GCE

Further Mathematics A

Y535/01: Additional Pure Mathematics

Advanced Subsidiary GCE

2020 Mark Scheme (DRAFT)

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Question			Answer	Marks	AO	Guidance
1	(a)		30 (mod 31) or –1 (mod 31)	**B1** **[1]**	**1.1**	**BC** No other answer to be accepted Note: 13 × 19 = 247 = 7 × 31 + 30 ≡ 30 (mod 31)
	(b)		$13x \equiv 9 \equiv 40 \equiv 71 \equiv \ldots \equiv 195$	**M1** **A1**	**1.1** **1.1**	Repeatedly adding 31s arriving at a multiple of 13
			so $x \equiv 15$ (mod 31) **OR** $x = 31n + 15$	**A1**	**2.2a**	$n \in \mathbb{Z}$ need not be stated
			Alternative method $13 \times 19 \equiv -1 \Rightarrow 13 \times (19\times13\times19) \equiv 1$ so $19\times13\times19 \equiv 12$ is the reciprocal of 13 (mod 31) Then $12 \times 13x \equiv 12 \times 9$ $\Rightarrow x \equiv 15$ (mod 31)	**M1** **M1** **A1**		Method for finding reciprocal (inverse) of 13 (mod 31) using **(a)** Multiplication by the reciprocal correct answer
				[3]		

Question			Answer	Marks	AO	Guidance
2	(a)		$xyh = 1000 \Rightarrow h = \dfrac{1000}{xy}$	B1	3.1b	
			$A = xy + 2xh + 2yh$	B1	1.1	**soi**
			$= xy + 2000\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$	M1 A1 **[4]**	2.1 1.1	Substitution of h expression from **(a) (i)** **AG** shown with supporting working
	(b)	(i)	$\dfrac{\partial A}{\partial x} = y + 2000\left(\dfrac{-1}{x^2}\right)$ and $\dfrac{\partial A}{\partial y} = x + 2000\left(\dfrac{-1}{y^2}\right)$	M1 A1 B1	1.1 1.1 1.1	Partially differentiating A w.r.t. x or y; either correct 2nd correct: **FT** 1st, with $x \leftrightarrow y$
			Both p.d.s set to zero and solving	M1	2.1	$x^2y = xy^2 = 2000$
			$x = y = 10 \times 2^{\frac{1}{3}}$	A1 **[5]**	1.1	Both correct
		(ii)	Substg. x, y back into formula for A; $300 \times 2^{\frac{2}{3}}$	M1 A1 **[2]**	1.1 1.1	Any exact equivalent e.g. $150 \times 2^{\frac{5}{3}}$, $75 \times 2^{\frac{8}{3}}$ **or** awrt 476 **BC**
3	(a)		13 divides each pair of digits of N (26, 13, 26, 52)	B1 **[1]**	2.4	Or applying a standard divisibility test
	(b)		$4 \mid 52$ (the final two digits of N) $\Rightarrow 4 \mid N$	B1	1.1	Applying these two divisibility tests
			$9 \mid$ digit-sum of N (= 27) $\Rightarrow 9 \mid N$	B1	1.1	
			Since hcf(4, 9) = 1, $4 \times 9 = 36 \mid N$	B1 **[3]**	2.4	Must explain that 4, 9 are co-prime as well as state the conclusion
	(c)		By *Euclid's Lemma*, $13 \mid 36 \times 725\,907$ and hcf(13, 36) = 1	M1	2.4	M for stating "Euclid's Lemma" (or full description of its result)
			$\Rightarrow 13 \mid 725\,907$	A1 **[2]**	2.2a	Clear outline of necessary conditions

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td></td><td>
<table>
<tr><th>$\times_{14}$</th><th>2</th><th>4</th><th>6</th><th>8</th><th>10</th><th>12</th></tr>
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</table>
</td><td>B1
B1
B1
B1
[4]</td><td>1.1
1.1
1.1
1.1</td><td>For any two lines (Rs or Cs) correct
For at least two Rs and two Cs correct
For LSP applying to complete table
For symmetry about main diagonal
(Must be fully correct for all 4 marks)</td></tr>
<tr><td></td><td>(b)</td><td></td><td>Closed since no other elements appear in the table
Identity is 8
Inverses: 6 is self-inverse
$2^{-1} = 4$ and $4^{-1} = 2$; $10^{-1} = 12$ and $12^{-1} = 10$
(Hence a group)</td><td>B1
B1
B1
B1
[4]</td><td>2.4
2.2a
1.2
2.5</td><td>Don't accept "closed, from table" only
Any clear indication of inverses (not just statement they exist)
That is, (2, 4) and (10, 12) are inverse-pairs
Associativity and conclusion not required</td></tr>
<tr><td></td><td>(c)</td><td>(i)</td><td>{8, 6} {8, 2, 4}</td><td>B1 B1
[2]</td><td>2.2a 1.1</td><td>One correct; both (and no extras). Ignore {8} and G</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>10, 12</td><td>B1 B1
[2]</td><td>1.1 1.1</td><td>One correct; both (and no extras)</td></tr>
<tr><td>5</td><td>(a)</td><td></td><td>Complementary Solution is $V_n = A \times 2^n$
For Particular Solution, try $V_n = an + b$
Then $V_{n+1} = 2V_n + n \Rightarrow an + (a + b) = 2an + 2b + n$
Comparing coefficients: $a = 2a + 1$ and $a + b = 2b$
$\Rightarrow a = b = -1$
General Solution is thus $V_n = A \times 2^n - n - 1$</td><td>B1
M1
A1
M1
A1
B1
[6]</td><td>1.2
1.1a
1.1
1.1
1.1
1.1</td><td>Allow $V_n = an$ for method mark
Substitution and comparing of coefficients
FT GS = CS + PS provided CS has one arbitrary constant and PS has none (and is a polynomial)</td></tr>
</table>

Question			Answer	Marks	AO	Guidance
5	(b)		$V_1 = 8 \Rightarrow A = 5$ so $V_n = 5 \times 2^n - n - 1$	M1	3.1a	soi (or **BC**)
			So $V_{20} = 5\,242\,859$	A1	1.1	accept exact value only.
				[2]		
6	(a)		$\mathbf{a} \times \mathbf{b} = -14\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$	B1	1.1	A correct vector product (possibly **BC**)
			Use of formula Area $\Delta = \frac{1}{2}\lvert \mathbf{a} \times \mathbf{b} \rvert$	M1	1.1	Including an attempt at a vector product
			Area $\Delta OAB = 5\sqrt{3}$	A1	1.1	Accept alternative exact equivalents (e.g. $\sqrt{75}$)
				[3]		
	(b)		$(\mathbf{r} - \mathbf{a}) \times (\mathbf{b} - \mathbf{a}) = \mathbf{0}$ is the line through A and B	M1	2.2a	
			so $\mathbf{c} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ or $\mathbf{c} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$	A1	3.1a	
			Area $\Delta OAC = \frac{1}{2}\lvert \mathbf{a} \times \mathbf{c} \rvert = \frac{1}{2}\lvert (1 - \lambda)\mathbf{a} \times \mathbf{a} + \lambda\, \mathbf{a} \times \mathbf{b} \rvert$	M1	2.1	From this point on, work may appear with numerical equivalent set-out
			$= \frac{1}{2}\lvert \mathbf{0} + \lambda\, \mathbf{a} \times \mathbf{b} \rvert$	M1	3.1a	Use of $\mathbf{a} \times \mathbf{a} = \mathbf{0}$
			Area $\Delta OAC = \frac{1}{2}$ Area $\Delta OAB \Rightarrow \lambda = \pm\frac{1}{2}$	A1	1.1	
			giving $\mathbf{c} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	A1	2.1	
			Alternative method			
			C is on the line AB	B1		
			Common "base" OA means that C is either the	M1		
			internal or the external bisector of AB	A1		(For half the "height")
				M1		At least one must be attempted
			i.e. $\mathbf{c} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ or $\frac{1}{2}(3\mathbf{a} - \mathbf{b})$	A1		
			giving $\mathbf{c} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	A1		Both correct
				[6]		

Question			Answer	Marks	AO	Guidance
7	(a)	(i)	E.g. $T_0 = 100\,000$ is the initial population as given $T_{k+1} = (1-r)\,T_k$ because a death-rate of r means that $1 - r$ of the population is left after each week. $0 \le k \le 12$ because the model given is only valid for twelve weeks.	B1 B1 B1 [3]	1.1 3.3 2.1	
		(ii)	$T_{12} = a^{12}\,T_0$ $1 - r = \sqrt[12]{0.00355} = 0.624\,96\ldots \Rightarrow r = 0.375$ to 3s.f.	M1 A1 [2]	3.1b 1.1	$a = r$ or $1 - r$ **AG**
	(b)	(i)	After 16 weeks, the number of frogs is $0.62496\ldots^{16} \times 100\,000 = 54.154\ldots$ So $54.154\ldots \times p \ge 30$ $\Rightarrow p \ge \dfrac{30}{54.154...} = 0.5539... = 0.554$ to 3 sf	B1 M1 A1 [3]	3.5c 3.1b 1.1	Allow use of 'T_{16}'. Or, starting again $0.62496\ldots^4 \times 355$ For 'their population' $\times p \ge 30$
		(ii)	**E.g.** The same weekly death-rate factor continues unchanged. The females will all lay eggs. Tadpoles instantly change to frogs and lay eggs at exactly the same time.	B1 [1]	3.3	
	(c)		**E.g.** 30 surviving females would produce 75000 eggs, so the population is smaller than it was to start with, so each 'round' will result in smaller and smaller populations.	B1 [1]	3.5a	No greater detail of analysis is required beyond "they would appear to be dying out so the figure of 30 in the model is not a good one"

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